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BRIEF COMMUNICATION

VOID-FRACTION RELATIONSHIPS FOR UPWARD FLOW OF SATURATED, STEAM-WATER MIXTURES

H. S. ISBIN and DAVID BIDDLE†
Department of Chemical Engineering and Materials Science, University of Minnesota, Minneapolis, MN 55455, U.S.A.

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INTRODUCTION

We would like to share our experiences in using void fraction correlations for the evaluation of the features of steam–water, upward flow. Our application was for solar boilers employing tubes with an i.d. of about 0.7 cm and an operating pressure of about 10 MPa. We started with the report by Butterworth (1975) who cast six models and correlations for predicting void fractions in concurrent gas–liquid flows into the following forms:

$$\frac{1-\bar{\alpha}}{\bar{\alpha}} = A\left(\frac{1-x}{x}\right)^p \left(\frac{\rho_G}{\rho_L}\right)^q \left(\frac{\mu_L}{\mu_G}\right)^r \qquad [1]$$

where $\bar{\alpha}$ is the void fraction averaged over the cross section, x is the quality, ρ is the density and μ is the viscosity; subscript G denotes the gas phase and subscript L denotes the liquid phase; and where the parameters A, p, q and r have the following values:

Correlation	A	p	q	r
Homogeneous Model	1	1	1	0
Zivi	1	1	0.67	0
Lockhart & Martinelli	0.28	0.64	0.36	0.07
Thom	1	1	0.89	0.18
Baroczy	1	0.74	0.65	0.13
Turner & Wallace	1	0.72	0.40	0.08

Butterworth cautiously suggested that [1] might be a useful form for correlating void fraction data.

Our interest was to consider additional correlations and to evaluate their applications for upward flow of steam–water mixtures, considering only saturated phases. Models applicable for subcooled boiling are not included in this review.

THE MARTINELLI-NELSON RESULTS

In the Butterworth presentation, the Lockhart and Martinelli correlation had been selected. For steam–water flows, the Martinelli–Nelson results should be used. We illustrate the use of

†Present address: Exxon Co., Baytown, TX 77520, U.S.A.

the Maurer (1960) tabulations covering a pressure range of 6.9–13.8 MPa. The physical properties of the saturated steam and water are functions of only the system pressure, and thus [1] may be written in the form:

$$\frac{1-\bar{\alpha}}{\bar{\alpha}} = \left(\frac{1-x}{x}\right)^{p} f(P). \qquad [2]$$

Empirical fits for p and $f(P)$ are

$$p = 0.6819 + (1.217 \times 10^{-2})P \qquad [3]$$

$$f(P) = \exp\,[(8.951 \times 10^{-2})P - 2.6439] \qquad [4]$$

for the pressure, P, varying from 6.9 to 13.8 MPa.

Another empirical fit, expressed more simply, is

$$\frac{1-\bar{\alpha}}{\bar{\alpha}} = 0.80\left(\frac{1-x}{x}\right)^{p} \left(\frac{\rho_G}{\rho_L}\right)^{0.60}. \qquad [5]$$

With this formulation, the Butterworth values for A is 0.80, q is 0.60, but the value for p retains a system pressure dependency as noted in [3].

MODELS COMPARED TO THE HOMOGENEOUS MODEL

For the upward flow of saturated steam–water mixtures, the homogeneous model for the void fraction represents the upper bound.

$$\frac{1-\bar{\alpha}_H}{\bar{\alpha}_H} = \left(\frac{1-x}{x}\right)\left(\frac{\rho_G}{\rho_L}\right) \qquad [6]$$

where $\bar{\alpha}_H$ is the void fraction of the homogeneous fluid. Correlations of the form

$$\bar{\alpha} = K\bar{\alpha}_H \qquad [7]$$

or

$$\bar{\alpha} = \bar{\alpha}_H - (\text{FACTORS}) \qquad [8]$$

have also been presented in the literature. The term K is equal or less than unity and might be expected to be dependent upon flow pattern, flow conditions and the physical properties of the phases.

The K factor approach starts with a physical model, and Bankoff (1960) proposed an analytical model for the bubble flow regime. He assumed a power law distribution for both velocity and void fraction profiles in a conduit and determined that K is a function of the exponents used. Reasonable choices of the power law exponents limit the values of K to lie between 0.6 and 1.0. For steam–water mixture flows, Bankoff concluded that a reasonable fit to the void fraction data could be achieved using

$$K = 0.71 + (1.5 \times 10^{-2})P \qquad [9]$$

where P is the system pressure in MPa.

Jones (1961) represented the pressure dependency by noting at the critical pressure that $\alpha = K = 1$. Also at $x = 1$, $K = 1$. The resulting form for K is

$$K = a + (1 - a)\bar{\alpha}^b \qquad [10]$$

where

$$a = 0.71 + 1.3 \times 10^{-2} P \qquad [11]$$

$$b = 3.53125 - (2.719 \times 10^{-2})P + (1.233 \times 10^{-2})P^2 \qquad [12]$$

and P is in MPa. The exponent b was fitted to follow the Martinelli–Nelson correlations.

Zuber (1965) took into account the radial distribution of the local void fraction and the local relative velocity. The resulting K factor is given by

$$K = \left(C_0 + \frac{\overline{\alpha v_{Gj}}}{\bar{\alpha} \cdot \bar{j}}\right)^{-1}. \qquad [13]$$

The distribution factor,

$$C_0 = \frac{\overline{\alpha j}}{\bar{\alpha} \cdot \bar{j}} \qquad [14]$$

accounts for the nonuniform radial distribution of the void fraction and is related to the inverse of the Bankoff multiplier. The second term, $\overline{\alpha v_{Gj}}/(\bar{\alpha} \cdot \bar{j})$, accounts for the nonuniform drift velocity. Values for C_0 and the weighted mean drift velocity, $\overline{\alpha v_{Gj}}/\bar{\alpha}$, are dependent upon flow regimes and have been determined, for example, by Zuber (1967) and Ishii (1977) for specified conditions. We believe that with improved characterization of flow regimes, such as discussed by Dukler & Taitel (1977), this model, including extensions into the subcooled boiling regions, will probably be more widely used.

COMPARISON OF MODELS

Table 1 includes the Butterworth correlations and correlations of the form given in [7] and [8]. The twelve correlations from table 1 are illustrated in figures 1–3 for qualities 0.05, 0.25 and 0.75 respectively for pressures from 0.7 to 13.8 MPa. For those correlations requiring inputs

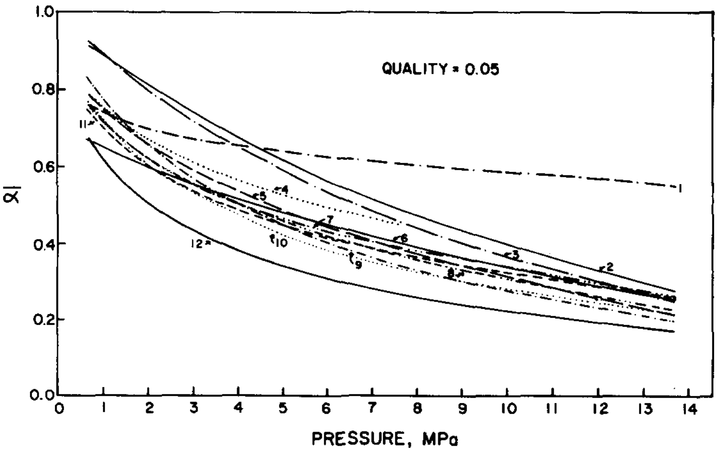

Figure 1. Steam–water void fraction predictions for 0.05 quality (see table 1 for correlations used).

Table 1. Void fraction correlations used in figures 1–3

(1) Lockhart & Martinelli [as given by Butterworth (1975)]

$$\frac{1-\bar{\alpha}}{\bar{\alpha}} = 0.28\left(\frac{1-x}{x}\right)^{0.64}\left(\frac{\rho_G}{\rho_L}\right)^{0.36}\left(\frac{\mu_L}{\mu_G}\right)^{0.07}.$$

(2) Homogenous

$$\frac{1-\bar{\alpha}}{\bar{\alpha}} = \left(\frac{1-x}{x}\right)\left(\frac{\rho_G}{\rho_L}\right).$$

(3) Moussali [as given by Friedel (1977)]

$$\bar{\alpha} = K\bar{\alpha}_H$$

$$K = 1 - \frac{(30.4/d) + 11}{60(1+1.6/d)(1+3.2/d)}$$

$$d = \frac{1-x}{x}\frac{\rho_G}{\rho_L}.$$

(4) Löscher [as given by Friedel (1977)]

$$\bar{\alpha} = \beta - \left(\frac{P}{P_c}\right)^{C_1}\beta^{C_2}(1-\beta)^{C_3}Fr_L{}^{C_4}\left(1-\frac{P}{P_c}\right)^{C_5}$$

where

$$C_1 = -0.22;\ C_2 = 1.39;\ C_3 = 0.8;\ C_4 = -0.25;\ C_5 = 3.4$$

$\beta = \bar{j}_G/(\bar{j}_G + \bar{j}_L)$, average volumetric flow concentration of the gas $= \bar{\alpha}_H$

$$\mathrm{Fr}_L = G^2/(\rho_L{}^2 g_c D_p).$$

(5) Hughmark (1962)

$$\bar{\alpha} = K\bar{\alpha}_H$$

$$z = \mathrm{Re}^{1/6}\mathrm{Fr}^{1/8}/y_L{}^{1/4}$$

where

$$\mathrm{Re} = \frac{D_p G}{(1-\bar{\alpha})\mu_L + \bar{\alpha}\bar{\mu}_G};\qquad \mathrm{Fr} = \frac{G^2\left[\frac{x}{\rho_G} + \frac{1-x}{\rho_L}\right]^2}{g_c D_p};$$

$$y_L = (\dot{m}_L/\rho_L)(\dot{m}_L/\rho_L + \dot{m}_G/\rho_G)$$

z	1.3	1.5	2.0	3.0	4.0	5.0	6.0	10	15	20	40	70	130
K	0.185	0.225	0.325	0.49	0.605	0.675	0.767	0.78	0.808	0.083	0.88	0.93	0.98.

(6) Bankoff (1960)

$$\bar{\alpha} = K\bar{\alpha}_H$$

$K = 0.71 + (1.5\times 10^{-2})P$ with P in MPa.

(7) Kütüçüğlu [as given by Friedel (1977)]

$$\bar{\alpha} = \beta - k(1-\beta)^{C_1}\mathrm{Fr}_L{}^{C_2}\left(1-\frac{P}{P_c}\right)^{C_3}$$

where

$$k = 1.0,\quad C_1 = 0.5,\quad C_2 = 0.2,\quad C_3 = 2.0.$$

(8) Yamazaki (1976)

$$\frac{\bar{\alpha}}{(1-\bar{\alpha})(1-h\bar{\alpha})} = \frac{\rho_L}{\rho_L}\frac{x}{1-x}$$

Table 1 (*Contd*).

where

$$h = (\bar{u}_G - \bar{u}_L)/\bar{j}_G = 1.0 \text{ for } E_0\lambda \geq 2\times 10^{-6}$$
$$= 0.57 \text{ for } E_0\lambda > 2\times 10^{-6}$$
$$E_0 = g_c D_p^2(\rho_L - \rho_G)/\sigma; \quad \lambda = \mu_L^2/\rho_L D_p \sigma.$$

(9) Thom [as given by Butterworth (1975)]

$$\frac{1-\bar{\alpha}}{\bar{\alpha}} = \left(\frac{1-x}{x}\right)\left(\frac{\rho_G}{\rho_L}\right)^{0.89}\left(\frac{\mu_L}{\mu_G}\right)^{0.18}$$

(10) Kowalczewski [as given by Friedel (1977)]

$$\bar{\alpha} = \beta - k(1-\beta)^{C_1}\mathrm{Fr}_L^{C_2}\left(1 - \frac{P}{P_c}\right)^{C_3}$$

where

$$k = 0.71, \quad C_1 = 0.5, \quad C_2 = -0.045, \quad C_3 = 1.0$$

see 4 for Fr_L.

(11) Baroczy [as given by Butterworth (1975)]

$$\frac{1-\bar{\alpha}}{\bar{\alpha}} = \left(\frac{1-x}{x}\right)^{0.74}\left(\frac{\rho_G}{\rho_L}\right)^{0.65}\left(\frac{\mu_L}{\mu_G}\right)^{0.13}.$$

(12) Zivi [as given by Butterworth (1975)]

$$\frac{1-\bar{\alpha}}{\bar{\alpha}} = \frac{1-x}{x}\left(\frac{\rho_G}{\rho_L}\right)^{0.67}.$$

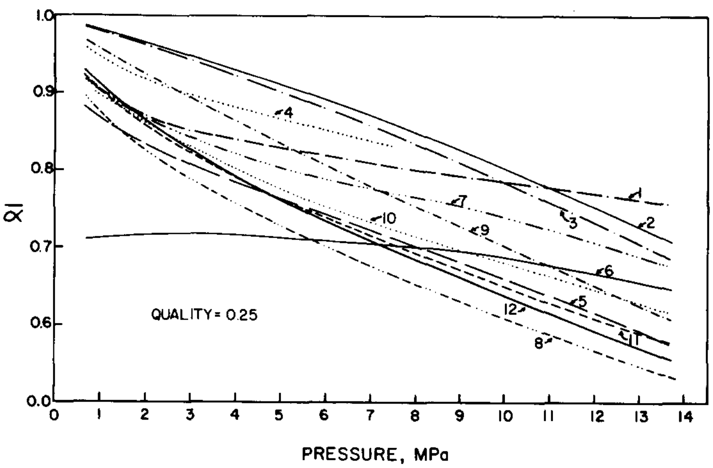

Figure 2. Steam–water void fraction predictions for 0.25 quality (see table 1 for correlations used).

other than just quality and pressure, specific values applicable to the solar boiler geometries and flow rates were selected. The purpose of the illustrations is not to dwell on the absolute values, but to indicate trends and spreads among correlations for even a specific application. Obviously other specific choices might alter some of the comparisons and thus we caution against formulating any generalizations.

Included in the representations is the Yamazaki (1976) correlation which is reported to have an accuracy of plus or minus 15% for both adiabatic and diabatic flow of steam–water mixtures

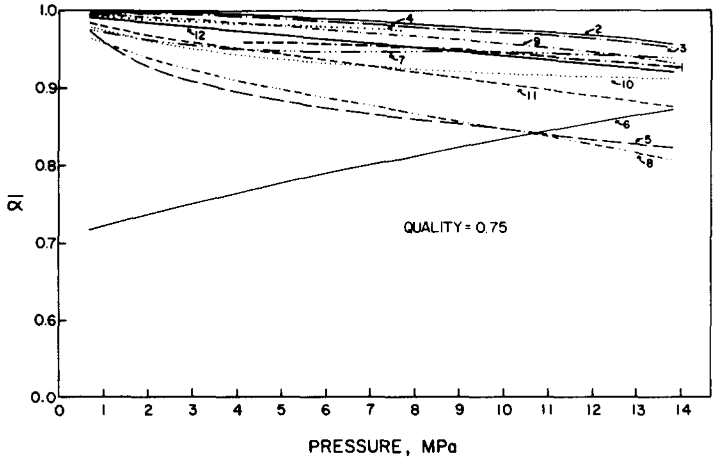

Figure 3. Steam–water void fraction predictions for 0.75 quality (see table 1 for correlations used).

and for pressures from atmospheric to 8.3 MPa. This large spread would encompass nearly all correlations given.

Judgment is required in choosing the correlation most applicable to the specific flow and geometrical conditions to be used. The use to which the void fraction is to be put is important. For example, emphasis may be directed to hold up, say for reaction rate applications, to voids for determining density effects, and to the evaluation of acceleration and frictional drops. The methods used for measuring the void fraction used to develop specific correlations may be a factor to be considered. Unfortunately, a void fraction data bank with thorough evaluations of the experimental errors and the system conditions is not available. Thus more meaningful evaluations of void fraction correlations have yet to be made.

NOMENCLATURE

C_0 Zuber distribution parameter
G mass flow rate of total mixture per unit area, $G_G + G_L$
G_G mass flow rate of gas per unit area
G_L mass flow rate of liquid per unit area
g_c gravitational proportionality constant
j total local volumetric flux
j_G local volumetric flux of gas phase
j_L local volumetric flux of liquid phase
K Bankoff factor to reduce homogeneous void fraction, including modifications
$\dot{m}_G$ mass flow rate of gas
$\dot{m}_L$ mass flow rate of liquid
P pressure
P_c critical pressure
S velocity ratio, $\bar{u}_G/\bar{u}_L$
s local velocity ratio, u_G/u_L
u_G local velocity of the gas phase
u_L local velocity of the liquid phase
v_{Gj} local drift velocity of gas phase, $u_G - j$
x quality, $\dfrac{\dot{m}_G}{\dot{m}_G + \dot{m}_L}$

Greek symbols

α local void fraction

$\bar{\alpha}$ void fraction averaged over cross section

$\bar{\alpha}_H$ void fraction for homogeneous model

β average volumetric flow concentration of gas, $\frac{\bar{j}_G}{\bar{j}_G + \bar{j}_L}$

μ_G viscosity of the gas phase

μ_L viscosity of the liquid phase

ρ_G density of the gas phase

ρ_L density of the liquid phase

σ surface tension

Additional symbols

$\bar{f}$ (bar over quantity) indicates average of a quantity over the cross section

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